On Supra $\alpha$– Connectedness in Supra Topological Spaces

Ghufran A. Abbas$^1$, Taha H. Jasim$^2$

$^1,^2$Department of Mathematics, College of Computer Science and Mathematics, Tikrit University, Tikrit, Iraq.

$^1$ghufranabas@gmail.com, $^2$tahahameed91@gmail.com

Abstract

The purpose of this paper is to introduce the concept called supra $\alpha$– connectedness in supra topological spaces and study some of the properties.

Keywords: supra topological space, supra $\alpha$– open set, supra $\alpha$– connected.

DOI: http://doi.org/10.32894/kujss.2019.14.4.1
1. Introduction:


In this paper, we introduces the concept of supra $\alpha$– connectedness and investigate about their relationships using the concept of continuity.

2. Preliminaries:

The aim of this paper is to study what is called the Supra $\alpha$– connectedness as well as the effect of some kinds of mapping on its. So we will need the following results and definitions.

**Definition 2.1.** [1] Let $X$ be a non-empty set. Let $\mu \subseteq P(X) = \{A : A \subseteq X\}$. Then $\mu$ is called a supra topology on $X$ if $\emptyset \in \mu, X \in \mu$. If $Y_\lambda \in \mu$ for every $\lambda \in \Lambda$ where $\Lambda$ is an arbitrary set, then $\bigcup_{\lambda \in \Lambda} Y_\lambda \in \mu$. The pair $(X, \mu)$ is called a supra topological space. Each element $A \in \mu$ is called a supra open set in $(X, \mu)$. The complement of $A$ is denoted by $A^c = X - A$, which is called a supra closed set in $(X, \mu)$.

**Definition 2.2.** [1] Let $(X, \mu)$ be a supra topological space. The supra closure of a set $A$ which is defined by supra– $cl(A) = \cap \{B \subseteq X : B$ is a supra closed set in $X$ such that $A \subseteq B\}$.

The supra interior of a set $A$ is denoted by supra– $Int(A)$ and is defined by supra– $Int(A) = \cup \{U \subseteq X : U$ is a supra open set in $X$ such that $U \subseteq A\}$.

**Definition 2.3.** [1] Let $(X, \mathcal{T})$ be a topological space and $\mu$ be a supra topology on $X$. We call $\mu$ a supra topology associated with $\mathcal{T}$ if $\mathcal{T} \subseteq \mu$.

**Definition 2.4.** [2] Let $(X, \mu)$ be a supra topological space. A subset $A$ of $X$ is called a supra $\alpha$– open set in $X$ if $A \subseteq supra \text{ Int}\left(supra \text{ cl}(supra \text{ Int}(A))\right)$ if $A$ is supra open set. The complement of supra $\alpha$– open set is called a supra $\alpha$– closed set.

**Definition 2.5.** [2] Let $(X, \mu)$ be a supra topological space. The supra $\alpha$– closure of a set $A$ is denoted by supra– $\alpha$– $cl(A)$, and is defined as follows:
Supra- $\alpha- cl(A) = \cap \{B \subseteq X : B$ is supra $\alpha-$ closed set in $X$ such that $A \subseteq B\}$. 

The supra $\alpha-$ interior of a set $A$ is denoted by supra- $\alpha-$ $Int(A)$, and is defined by supra- $\alpha-$ $Int(A) = \cup \{U \subseteq X : U$ is supra $\alpha-$ open set in $X$ such that $U \subseteq A\}$. Clearly it is obvious that supra- $\alpha-$ $cl(A)$ is a supra $\alpha-$ closed set. In the same way, supra- $\alpha-$ $Int(A)$ is supra $\alpha-$ open set.

Throughout this paper, $(X, \mathcal{T})$ and $(Y, \mathcal{T}^*)$ will denoted topological spaces by the researchers. Where $\mu$ and $\mu^*$ will be their associated supra topologies with $\mathcal{T}$ and $\mathcal{T}^*$ respectively is that $\mathcal{T} \subseteq \mu$ and $\mathcal{T}^* \subseteq \mu^*$.

**Theorem 2.6**.[2]. Let $(X, \mu)$ be a supra topological space. Then every supra open set in $X$ is supra $\alpha-$ open set in $X$.

The converse of the theorem (2.6) need not be true as shown by the following example.

**Example 2.7**.[2]. Suppose $X = \{a, b, c\}$ and have the supra topology $\mu = \{\emptyset, X, \{a\}\}$. The set $\{a, b\} \notin \mu$, so the set $\{a, b\}$ is not supra open set in $(X, \mu)$. Now since it clearly follows that supra- $\alpha-$ $Int[\supra-\alpha-cl[\supra-\alpha-\mathcal{I}(\{a, b\})]] = \supra-\alpha-\mathcal{I}[\supra-\alpha-cl(\{a\})] = \supra-\alpha-\mathcal{I}((X)] = X$. Therefore it follows that $\{a, b\}$ is a supra $\alpha-$ open set in $(X, \mu)$.

**Definition 2.8**.[2]. A function $f: (X, \mu) \to (Y, \mu^*)$ is called a supra $\alpha-$ continuous function if the inverse image of each supra open set in $Y$ is a supra $\alpha-$ open set in $X$.

**Definition 2.9**[3] A function $f: (X, \mu) \to (Y, \mu^*)$ is called $i-$ supra $\alpha-$ continuous if $f^{-1}(Y)$ of each supra $\alpha-$ open subset of $Y$ is supra $\alpha-$ open subset of $X$.

**Definition 2.10**[3] A function $f: (X, \mu) \to (Y, \mu^*)$ is called strongly supra $\alpha-$ continuous if the inverse image of every supra $\alpha-$ open subset of $Y$ is supra open in $X$.

**Definition 2.11**[3] A function $f: (X, \mu) \to (Y, \mu^*)$ is called perfectly supra $\alpha-$ continuous if the inverse image of every supra $\alpha-$ open subset of $Y$ is both supra open and supra closed in $X$.

**Definition 2.12**[3] A function $f: (X, \mu) \to (Y, \mu^*)$ is called totally supra $\alpha-$ continuous if the inverse image of every supra open set in $Y$ is both supra $\alpha-$ closed and supra $\alpha-$ open in $X$. 

Web Site: www.uokirkuk.edu.iq/kujss E-mail: kujss@uokirkuk.edu.iq, kujss.journal@gmail.com
3. Supra $\alpha$– Connected:

**Definition 3.1.[4]** A supra topological space $(X, \mu)$ is said to be supra connected if $X$ cannot be written as a disjoint union of two non-empty supra open subsets of $X$. A subset of $(X, \mu)$ is supra connected if it is supra connected a subspace.

**Definition 3.2** A supra topological space $(X, \mu)$ is said to be supra $\alpha$– connected if $X$ cannot be written as disjoint union of two non-empty supra $\alpha$– open sets. A subset of $(X, \mu)$ is supra $\alpha$– connected if it is supra $\alpha$– connected as subspace.

**Theorem 3.3**
Every supra connected is supra $\alpha$– connected space.

**Proof:** Let $X$ be supra connected. To show that $X$ is supra $\alpha$– connected. Since $X$ is supra connected. Then $X \neq A \cup B$ where $A$ and $B$ are disjoint non-empty supra open sets. Then by Theorem (2.6), we have $A$ and $B$ are disjoint non-empty supra $\alpha$– open sets. Thus $X$ is supra $\alpha$– connected.

**Theorem 3.4**
If $f: (X, \mu) \to (Y, \mu')$ be a surjective and supra $\alpha$– continuous mapping. Let $X$ be supra $\alpha$– connected, then $Y$ is supra connected.

**Proof:**
Suppose $Y$ is not supra connected.

Let $Y = A \cup B$, where $A$ and $B$ are disjoint non-empty supra open sets in $Y$. Since $f$ is supra $\alpha$– continuous surjective map, then $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty supra $\alpha$– open sets in $X$. This contradicts the fact that $X$ is supra $\alpha$– connected. Hence $Y$ is supra connected.

**Theorem 3.5**
Let $f: (X, \mu) \to (Y, \mu')$ be a surjective and strongly supra $\alpha$– continuous mapping. Let $X$ be supra connected space. Then $Y$ is supra $\alpha$– connected space.

**Proof:** Suppose $Y$ is not supra $\alpha$– connected. Let $Y = A \cup B$, where $A$ and $B$ are disjoint non-empty supra $\alpha$– open sets in $Y$. 
Since $f$ is strongly supra $\alpha$– continuous surjective map, then $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty supra open sets in $X$. This contradicts the fact that $X$ is supra connected. Hence $Y$ is supra $\alpha$– connected.

**Theorem 3.6**

If $f: (X, \mu) \rightarrow (Y, \mu^*)$ be a surjective and perfectly supra $\alpha$– continuous map and $X$ is supra connected, then $Y$ is supra $\alpha$– connected .

**Proof:** Suppose $Y$ is not supra $\alpha$– connected. Let $Y = A \cup B$, where $A$ and $B$ are disjoint non-empty supra $\alpha$– open sets in $Y$. Since $f$ is perfectly supra $\alpha$– continuous surjective map, then $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty supra open sets and supra closed set in $X$. This contradicts the fact that $X$ is supra connected. Hence $Y$ is supra $\alpha$– connected .

**Theorem 3.7**

If $f: (X, \mu) \rightarrow (Y, \mu^*)$ be a surjective and $i$– supra $\alpha$– continuous map and $X$ is supra $\alpha$– connected, then $Y$ is supra $\alpha$– connected .

**Proof:** Suppose $Y$ is not supra $\alpha$– connected. Let $Y = A \cup B$, where $A$ and $B$ are disjoint non-empty supra $\alpha$– open sets in $Y$. Since $f$ is $i$– supra $\alpha$– continuous surjective map, then $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty supra $\alpha$– open sets in $X$. This contradicts the fact that $X$ is supra $\alpha$– connected. Hence $Y$ is supra $\alpha$– connected.

**Theorem 3.8**

If $f: (X, \mu) \rightarrow (Y, \mu^*)$ be a surjective and totally supra $\alpha$– continuous map and $X$ is supra $\alpha$– connected, then $Y$ is supra connected.

**Proof:** Suppose $Y$ is not supra connected. Let $Y = A \cup B$, where $A$ and $B$ are disjoint non-empty supra open sets in $Y$. Since every supra open set is supra $\alpha$– open set, $A$ and $B$ are disjoint non-empty supra $\alpha$– open set in $Y$. Since $f$ is totally supra $\alpha$– continuous surjective map, $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-
empty is both supra $\alpha$– open and supra $\alpha$– closed in $X$. This contradicts the fact that $X$ is supra $\alpha$– connected. Hence $Y$ is supra connected.

**Theorem 3.9**

Let $C$ and $D$ be subsets of a supra topological space $X$. Assume that $C$ is supra $\alpha$– connected and $C \subseteq D$. Further assume that $U$ and $V$ form a separation of $D$ in $X$. Then either $C \subseteq U$ or $C \subseteq V$.

**Proof:** Suppose that neither $C \subseteq U$ nor $C \subseteq V$. Then $U \cap C \neq \emptyset$ and $V \cap C \neq \emptyset$. It follows that $U$ and $V$ form a separation of $C$ in $X$, contradicting that $C$ is supra $\alpha$– connected. Therefore either $C \subseteq U$ or $C \subseteq V$.

**Theorem 3.10**

Let $C$ be a supra $\alpha$– connected subspace in $X$, and assume that $C \subseteq A \subseteq$ supra $\alpha$– $cl(C)$, then $A$ is also supra $\alpha$– connected.

**Proof:** Suppose that $A$ is not supra $\alpha$– connected in $X$, and let $U$ and $V$ form a separation of $A$ in $X$. Then by 3.9, either $C \subseteq U$ or $C \subseteq V$. We may assume, without loss of generality, that $C \subseteq U$. Hence $C \cap V = \emptyset$. But, since $U$ and $V$ form a separation of $A$ in $X$, it follows that $A \cap V \neq \emptyset$. Pick $x \in A \cap V$. Now, $x \in A$ and $A \subseteq$ supra $\alpha$– $cl(C)$ imply $x \in$ supra $\alpha$– $cl(C)$. But $x \in V$, an open set in $X$ which is disjoint from $C$. So $x$ cannot be in the supra $\alpha$– closure of $C$, yielding a contradiction. Thus, it follows that $A$ is supra $\alpha$– connected.

**References:**


